Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8998216112

ADDITIONAL MATHEMATICS **4037/21**

Paper 2 **May/June 2021**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 **(a)** Write the expression $x^2 - 6x + 1$ in the form $(x+a)^2 + b$, where a and b are constants. [2]

(b) Hence write down the coordinates of the minimum point on the curve $y = x^2 - 6x + 1$. [1]

2 Variables x and y are such that, when $\ln y$ is plotted against $\ln x$, a straight line graph passing through the points (6, 5) and (8, 9) is obtained. Show that $y = \mathrm{e}^p x^q$ where p and q are integers. [4]

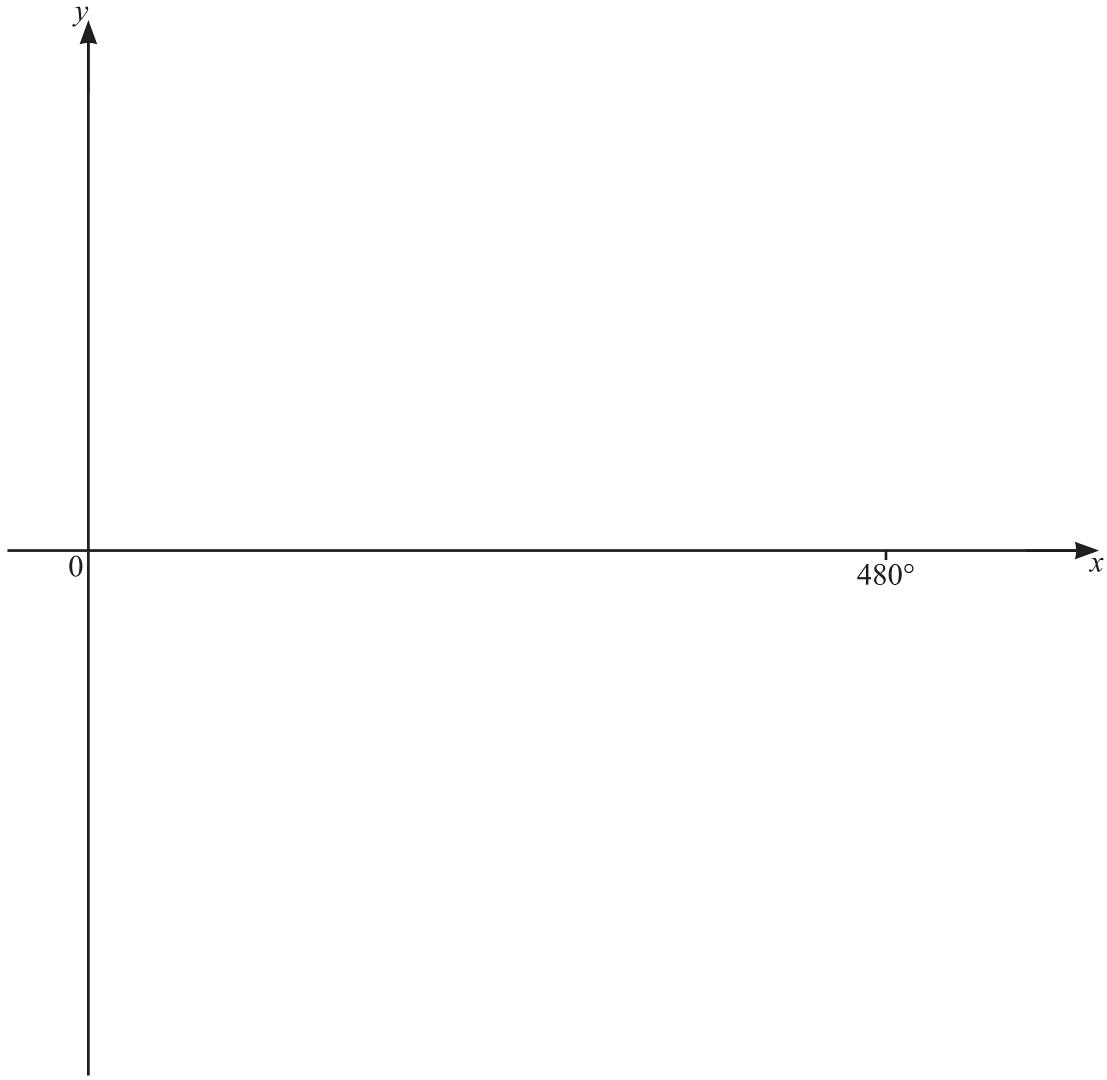
3 **(a)** Solve the inequality $|4x-1|>9$. [3]

(b) Solve the equation $2x-11\sqrt{x}+12=0$. [3]

4 The graph of $y = a + 2\tan bx$, where a and b are constants, passes through the point $(0, -4)$ and has period 480°.

(a) Find the value of a and of b. [3]

(b) On the axes, sketch the graph of y for values of x between 0° and 480°. [2]



5 The curves $y = x^2$ and $y^2 = 27x$ intersect at $O(0, 0)$ and at the point A. Find the equation of the perpendicular bisector of the line OA. [8]

6 Variables x and y are such that $y = \mathrm{e}^{\frac{x}{2}} + x\cos 2x$, where x is in radians. Use differentiation to find the approximate change in y as x increases from 1 to $1+h$, where h is small. [6]

7 Find the exact values of the constant k for which the line $y = 2x + 1$ is a tangent to the curve $y = 4x^2 + kx + k - 2$. [6]

8 In this question, a, b, c and d are positive constants.

(a) **(i)** It is given that $y = \log_a(x+3) + \log_a(2x-1)$. Explain why x must be greater than $\frac{1}{2}$. [1]

(ii) Find the exact solution of the equation $\dfrac{\log_a 6}{\log_a(y+3)} = 2$. [3]

(b) Write the expression $\log_a 9 + (\log_a b)(\log_{\sqrt{b}} 9a)$ in the form $c + d\log_a 9$, where c and d are integers. [4]

9 A curve is such that $\frac{d^2y}{dx^2} = \sin\left(6x - \frac{\pi}{2}\right)$. Given that $\frac{dy}{dx} = \frac{1}{2}$ at the point $\left(\frac{\pi}{4}, \frac{13\pi}{12}\right)$ on the curve, find the equation of the curve. [7]

10 Relative to an origin O, the position vectors of the points A, B, C and D are

$$\overrightarrow{OA} = \begin{pmatrix} 6 \\ -5 \end{pmatrix},\ \overrightarrow{OB} = \begin{pmatrix} 10 \\ 3 \end{pmatrix},\ \overrightarrow{OC} = \begin{pmatrix} x \\ y \end{pmatrix} \text{ and } \overrightarrow{OD} = \begin{pmatrix} 12 \\ 7 \end{pmatrix}.$$

(a) Find the unit vector in the direction of $\overrightarrow{AB}$. [3]

(b) The point A is the mid-point of BC. Find the value of x and of y. [2]

(c) The point E lies on OD such that $OE : OD$ is $1 : 1 + \lambda$. Find the value of λ such that $\overrightarrow{BE}$ is parallel to the x-axis. [3]

11 The 2nd, 8th and 44th terms of an arithmetic progression form the first three terms of a geometric progression. In the arithmetic progression, the first term is 1 and the common difference is positive.

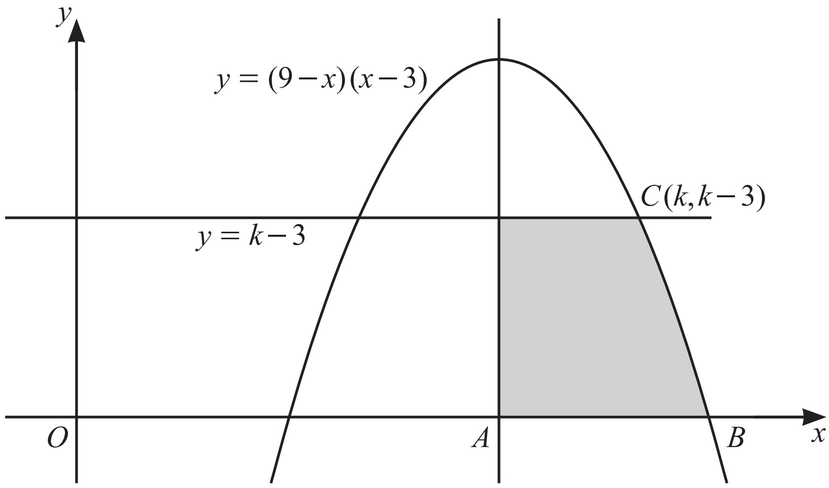
(a) **(i)** Show that the common difference of the arithmetic progression is 5. [5]

(ii) Find the sum of the first 20 terms of the arithmetic progression. [2]

(b) **(i)** Find the 5th term of the geometric progression. [2]

(ii) Explain whether or not the sum to infinity of this geometric progression exists. [1]

12



The diagram shows part of the curve $y = (9 - x)(x - 3)$ and the line $y = k - 3$, where $k > 3$. The line through the maximum point of the curve, parallel to the y-axis, meets the x-axis at A. The curve meets the x-axis at B, and the line $y = k - 3$ meets the curve at the point $C(k,\ k - 3)$. Find the area of the shaded region. [9]

Continuation of working space for Question 12.

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